

The Cauchy horizon in Higher-derivative Gravity Theories

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Abstract

A class of exact solutions of the field equations with higher derivative terms is presented when the matter field is a pressureless null fluid plus a Maxwellian static electric component. It is found that the stable solutions are black holes in anti de Sitter background. The issue of the stability of the Cauchy horizon is discussed.

I. INTRODUCTION

Why should one wish to study the internal structure of a black hole? Aside from sheer curiosity there is the intellectual challenge of testing our present physical theories up to their extreme boundary of validity.

In fact the spacetime outside a black hole is indeed a rather uninteresting subject. Thanks to the no-hair theorem we know the late time structure of the spacetime after the star has radiated away all the asphericities [1]. Instead, the ultimate fate of the star that undergoes a gravitational collapse, is still an open issue.

The plausible, still unproven strong cosmic censorship conjecture states [2] that the singularity at $r = 0$ is spacelike. The spacetime near the singularity can probably be described by the general mixmaster type solution [3].

It is however puzzling that even an infinitesimally small amount of angular momentum generates the Kerr-Newman singularity which is instead timelike. In this case the situation is perhaps more dramatic due to the presence of a null hypersurface, the Cauchy horizon (CH), boundary of predictability of the evolution of the fields. Like in the Reissner-Nordström solution, the existence of this null hypersurface makes the existence of a timelike singularity in a region beyond it physically irrelevant.

As it was pointed out by Penrose, [2] the Cauchy horizon is also a surface of infinite blue-shift. A free-falling observer crossing the Cauchy horizon will measure in-falling radiation to have infinite energy density. When only ingoing radiation is present a weak non-scalar

singularity forms, which is classified as a whimper singularity [4]. Whimper singularities are unstable to perturbations that can transform them into stronger, scalar, singularities.

This scenario can be examined more closely by assuming spherical symmetry. When an outflow crossing the Cauchy horizon is included in the analysis, the (Bondi) mass function is found to diverge exponentially [5] at late advanced times v

$$m \sim e^{\kappa v} \quad (1)$$

where κ is the surface gravity of the inner horizon located at $v = +\infty$. The only non-vanishing component of the Weyl tensor, the invariant Ψ_2 , is proportional to the mass function and a scalar curvature singularity forms along the Cauchy horizon [6,7]. This phenomenon, named “mass inflation”, has become a topic of increasing interest and lot of effort is devoted to study this subject [8].

The Reissner-Nordström black hole shares the same causal structure as the Kerr-Newman black hole, so it is not surprising that the basic picture derived in spherical symmetry should be qualitatively the same for the more general black hole. Some investigations suggest that the general scenario derived for spherical symmetry does not change dramatically in a non-spherical black hole [9–11]. In fact one key result of the mass inflation picture is that the rate of the exponential divergence of the mass function is entirely determined by the surface gravity which is constant along the generators of the Cauchy horizon.

The relevant question is then to understand how much the classical description of the spacetime near the singularity changes when quantum effects are taken into account. In particular, would quantum effects enforce the strong censorship conjecture? A first analysis has been already performed in [12] where the semi classical approach has been used to show that the classical picture remains valid up to few Plank lengths from the CH. The mass function at that “time” has already grown up to the mass of the observable Universe!

In fact, the classical description of the black hole interior is simplified by causality: behind the event horizon the coordinate r is timelike, so a descent into a black hole is a progression in time. The evolution down to any particular radius is only influenced by the initial data at larger radii. For this reason, a mean field, semi-classical description of quantum effects cannot drastically change the classical picture. This scenario can instead change in two-dimensional models, as shown by [13].

However, there is no reason why one should assume that a perturbative calculation can be applied when the curvature has reached planckian level. Only in a framework of a quantum gravity calculation one could hope to address this issue.

In recent years it has become clear that a very convenient way to quantise gravity is to describe it as a quantum effective field theory [14–16]. As a result one is lead to consider a more general action that is a functional of any geometrical invariant \mathfrak{R} which can be constructed from the principle of general covariance, $\mathfrak{R} = \{R, R_{\alpha\beta}R^{\alpha\beta}, C_{\alpha\beta\gamma\delta}C^{\alpha\beta\gamma\delta}, \dots\}$, and of the matter field ϕ and its derivatives

$$S[\phi, g] = \int d^4x \sqrt{-g} \mathcal{L}(\mathfrak{R}, \phi, \nabla^2 \mathfrak{R}, \nabla^2 \phi, \nabla^{2j} \mathfrak{R}, \nabla^{2j} \phi, \dots) \quad (2)$$

This new theory is analogous to the Euler-Heisenberg Lagrangian, low-energy effective theory of the QED. The important point is that both QED and the Euler-Heisenberg theory represent *the same theory* at different scale lengths. One would not include those new operators in the QED Lagrangian since they are suppressed as inverse power of the ultraviolet cutoff in the infrared region showing their “irrelevant” character in this case. They can become instead significant if we move the original cut-off of the theory in the infrared region, where the theory must describe phenomena at much lower energy scales.

This situation is similar in gravity. Due to the smallness of the ratio between the Newton constant and the Fermi constant, modifications of Einstein’s theory as in eq. (2), are experimentally indistinguishable from the standard theory at ordinary energy scales [17]. However at higher energy scales it can alter the physics content of the theory in a significant way. They might eventually destroy the unitarity of the S-matrix, signalling that a new physics sets in at some energy scale. The theory is perfectly well defined below that threshold, and it makes precise predictions. This happens also in the so-called renormalizable theories. The Standard Model, for instance, has to be regarded as an effective field theory which breaks down at the TeV scale, where the Higgs would enter in a strongly self-coupled phase.

Higher-derivative gravity theories arise also through the coupling of a quantised field with the classical background geometry [18] through the renormalization process. They are general functions of \mathfrak{R} and are needed to cancel the ultraviolet divergences for any given order in perturbation theory.

However, the standard perturbative approach based on the scaling property of the Green’s functions under a rescaling of the metric [19] handles only a finite number of operators, those which are important for the ultraviolet fixed point. In this way one cannot follow the evolution of the irrelevant, i.e. non-renormalizable operators. A coupling is irrelevant, marginal or relevant if it, respectively, it gets smaller, it does not run, it grows when the cut-off is lowered from the ultraviolet towards the infrared [20]. The irrelevant operators, even if they are not present in the bare Lagrangian, mix their evolution with the renormalised trajectory of the relevant couplings. Although near the Gaussian ultraviolet fixed point their running is suppressed, they can behave in a quite different manner in other scaling region and they can eventually drive the system in a different continuum limit.

It is therefore important to trace the evolution of all the coupling constants generated by the renormalization procedure in order to study the phase structure of the theory. In this way one recovers the predictability power of the theory by retaining only few relevant operators at a given fixed point. If for example a new scale other than the cut-off is present in the problem, the scaling laws change near that scale and the standard ultraviolet relevant interactions might not be sufficient to describe the physics at a crossover between ultraviolet and infrared.

The renormalization group approach used in Statistical Mechanics is the best tool to study problems where many scales are coupled together [21]. A powerful way of obtaining a differential form of the RG transformation has been formulated by Wegner and Houghton [22]. Starting from a bare action S_k at the cut-off k one first calculates $S_{k-\Delta k}$ in

$$e^{-S_{k-\Delta k}[\phi]} = \int \mathcal{D}[\psi] e^{-S_k[\phi+\psi]} \quad (3)$$

by using the loop expansion, where ψ and ϕ respectively have non-zero Fourier components only in the momentum shells $k - \Delta k < p \leq k$ and $p \leq k - \Delta k$. The differential RG transformation is obtained by taking the limit of an infinitesimal shell $\Delta k/k \rightarrow \delta k/k$. The higher loop contributions in Eq. (3) are suppressed as powers of $\Delta k/k$ in the limit $\Delta k \rightarrow 0$ for finite k and an exact, non-perturbative, one-loop RG equation is obtained

$$k \frac{dS_k[\phi]}{dk} = -\frac{1}{2} \langle \ln \frac{\delta^2 S[\phi]}{\delta \phi^2} \rangle + \langle \frac{\delta S[\phi]}{\delta \phi} \left(\frac{\delta^2 S[\phi]}{\delta \phi^2} \right)^{-1} \frac{\delta S[\phi]}{\delta \phi} \rangle \quad (4)$$

where the brackets indicates the sum over the Fourier components within the shell. This functional equation rules the evolution of all the interaction terms that are generated in the renormalization procedure. It has been applied in [23] to discuss the evolution of higher-derivative (HD) operators generated by the inflation field. In particular it has been shown that even if they are not present at the cut-off - which has been chosen below the Planck scale! - they show up above the mass threshold and influence the renormalised flow.

A consistent quantum gravity calculation in the framework of the effective theories, must be performed by including the HD operators from the beginning [16]. Within a perturbative (weak field) calculation one can use the Einstein-Hilbert truncation [24], but in a strongly non perturbative situation one should consider a more general Lagrangian and, with the help of the renormalization group equations, look for the consistent field configurations that effectively dominate the path integral at that scale of energy.

The first step towards this goal is to understand the tree-level structure of the space-time in the interior of a black hole, when higher-derivative operators are included from the beginning.

Those new operators can produce new instantons configuration in the path integral formulation of quantum gravity, with stronger weight than the solutions of the standard Einstein theory. If one wants to perform a saddle point evaluation of the path integral the first step is to determine the stationary points of the (Euclidean) action. Expansion around those saddle points are performed by writing

$$\gamma_{\mu\nu} = g_{\mu\nu} + h_{\mu\nu} \quad (5)$$

where, as usual, treating $g_{\mu\nu}$ as a background field and $h_{\mu\nu}$ as a quantum field one generates the usual perturbation expansion that can be expressed in terms of Feynman diagrams. The strong assumption that is usually believed, is that vacuum state for the renormalised system is still given by the $g_{\mu\nu}$ field configuration. Is this assumption still valid in the renormalised system when new higher dimensional operators are generated by the renormalization process? If not, we have chosen a wrong, unstable, saddle point. Strominger has shown [25] that for quadratic gravity, the Minkowsky solution is the only one having zero ADM energy. However that is not necessarily true for a more general non-linear gravity theory. As it has recently been pointed out by [26] where a completely new vacuum structure has appeared already for $R + R^3$ a Lagrangian.

In this work we shall reduce the original HD theory to the standard second order form by means of the method outlined in [26,27]. We shall obtain an equivalent (locally isomorphic) second order theory with an additional, non-geometric degree of freedom represented by a non-ghostlike scalar field. We call “vacuum” a solution which is stable with respect to the elementary excitation of this new field. We show that for a general non-linear Lagrangian the theory has non-trivial vacuum solutions representing BH embedded in Anti de Sitter spacetime.

We shall also comment on the stability of the Cauchy horizon in those solutions.

II. BASIC EQUATIONS

Let us consider the following action

$$S = \int d^4x \sqrt{-g} (l_g(R) + l_m(\mathcal{A}, g)) \quad (6)$$

where $l_g(R)$ is a non-linear function of the scalar curvature R which we shall suppose to be an analytic function of R

$$l_g(R) = \sum_n^{\infty} \frac{\epsilon_n}{n!} R^n \quad (7)$$

ϵ_n are the coupling constants of the generic power of R and $l_m(\mathcal{A})$ is the Lagrangian density of the matter field \mathcal{A} . We use the signature $(-+++)$ and we set $c = \hbar = 8\pi G$. With these units $\epsilon_0 = -\Lambda$ corresponds to the vacuum energy, being Λ the cosmological constant and $\epsilon_1 = 1/2$. The gravitational field equations for such a system are the following fourth order differential equations

$$\frac{\delta}{\delta g_{\mu\nu}} [\sqrt{-g} l_g(R)] = l'_g(R) R_{\mu\nu} - \frac{1}{2} l_g(R) g_{\mu\nu} - \nabla_\mu \nabla_\nu l'_g(R) + g_{\mu\nu} \square l'_g(R) = T_{\mu\nu}(\mathcal{A}, g) \quad (8)$$

where $T_{\mu\nu}$ is the stress-energy tensor of the matter field which obeys $\nabla_\mu T^{\mu\nu} = 0$.

In order to analyse the above theory it is convenient [26] to introduce an auxiliary field ψ so that the action now reads

$$S = \int d^4x \sqrt{-g} (l'_g(R)(R - \psi) - l_g(\psi) + l_m(A, g)) \quad (9)$$

with the equation of motion

$$l''_g(\psi)(R - \psi) = 0. \quad (10)$$

The new action, written with the help of the auxiliary field, is (at the classical level) equivalent to the original theory provided that we are not at the critical points $l''_g(R) = 0$, but it is not of the second order form. In order to reduce the action (10) in a canonical second order form, one introduces a pair of new variables $(\bar{g}_{\mu\nu}, \omega)$ related to $g_{\mu\nu}$ by

$$e^\omega = l'_g(R); \quad \bar{g}_{\mu\nu} = e^\omega g_{\mu\nu} \quad (11)$$

where $l'_g > 0$ is assumed and $\Psi(\omega)$ becomes a solution (not necessarily unique), of

$$l'_g(\Psi(\omega)) - e^\omega = 0. \quad (12)$$

This transformation has already been used in the literature, and it has been generalised to the case of Lagrangians depending on $\nabla^{2k} R$ (see [27] for a general review on the subject). Under the above conformal transformation the Ricci scalar density transforms like

$$\sqrt{-g}R = \sqrt{-\bar{g}}e^{-\omega} \left(\bar{R} - \frac{3}{2}(\bar{\nabla}\omega)^2 - 3\bar{\nabla}^2\omega \right). \quad (13)$$

By dropping a total derivative term, the action has been reduced to the canonical form

$$S = \int d^4x \sqrt{-\bar{g}} \left(\bar{R} - \frac{3}{2}\bar{g}^{\mu\nu}\partial_\mu\omega\partial_\nu\omega - 2V(\omega) + e^{-2\omega}l_m(\mathcal{A}, e^{-\omega}\bar{g}) \right) \quad (14)$$

where the potential term

$$V(\omega) = \frac{1}{2}e^{-2\omega} \left(e^\omega\psi(\omega) - l_g(\psi(\omega)) \right). \quad (15)$$

depends on the specific original higher-derivative theory. The field equations thus read

$$\bar{G}_{\mu\nu} = t_{\mu\nu} + \bar{T}_{\mu\nu} \quad (16)$$

where $\bar{G}_{\mu\nu}$ is the Einstein tensor of the reduced theory

$$\bar{G}_{\mu\nu} = \bar{R}_{\mu\nu} - \frac{1}{2}\bar{g}_{\mu\nu}\bar{R} \quad (17)$$

$t_{\mu\nu}$ is the stress energy-tensor of the scalar field

$$t_{\mu\nu} = \frac{3}{2} \left[\partial_\mu\omega\partial_\nu\omega - \frac{1}{2}\bar{g}_{\mu\nu}\bar{g}^{\alpha\beta}\partial_\alpha\omega\partial_\beta\omega \right] - \bar{g}_{\mu\nu}V(\omega) \quad (18)$$

and $\bar{T}_{\mu\nu}$ an effective stress-energy tensor generated by the interaction between the original matter field and the non-geometric gravitational new degrees of freedom represented by the scalar ω ,

$$\bar{T}_{\mu\nu} = e^{-\omega}T_{\mu\nu}(\mathcal{A}, e^{-\omega}\bar{g}). \quad (19)$$

One should also note that in general the two terms on the left hand side of the equation (16) are not separately conserved, but it holds instead the conservation law $\nabla_\mu \bar{G}^{\mu\nu} = 0$. We stress that the spin-0 field introduced with the help of eq.(11) has physical meaning since it corresponds to an additional degree of freedom already present in the starting non-linear Lagrangian. Therefore the advantage of having used this approach is that this new degree of freedom has been now made explicit in the reduced Lagrangian.

Since we are interested in studying the structure of the CH in the presence of HD terms, we consider a system where the gravitational field is coupled with a pressureless null fluid plus an electromagnetic contribution coming from a static electric field generated by a charge of strength e . The stress-tensor for the matter field is therefore given by

$$T_{\mu\nu} = \rho l_\mu l_\nu + E_{\mu\nu} \quad (20)$$

where $E_\mu^\nu = e^2/8\pi r^4 \text{diag}(1, 1, -1, -1)$ is the Maxwellian component of the electric field, ρ is the energy density of the radiation and l_μ is a null vector tangent to the radial null geodesic. In the following we shall specify our calculations in the case of spherical symmetry. One can introduce a null tetrad $\{l_\mu, n_\mu, m_\mu, \bar{m}_\mu\}$ with $l_\mu n^\mu = -1 = -m_\mu \bar{m}^\mu$. The metric tensor then reads

$$g_{\mu\nu} = 2[\bar{m}_{(\mu} m_{\nu)} - l_{(\mu} n_{\nu)}]. \quad (21)$$

and

$$E_{\mu\nu} = \frac{e^2}{4\pi r^4} [l_{(\mu} n_{\nu)} + \bar{m}_{(\mu} m_{\nu)}] \quad (22)$$

In our case the matter Lagrangian is conformally invariant, and the equation of motion for the matter field completely decouple from those for the scalar field. We analyse the case of $\omega = \text{const.}$ and the solutions of the equation of motion correspond to constant field configuration that are extrema for the potential

$$\frac{dV}{d\omega} = 0 \quad (23)$$

The vacuum case -no matter field- has been analysed in [26]. In our case we see that the conformal transformation in (11) generates a conformal rescaling of the null tetrad in (21) with $l_\mu \rightarrow \tilde{l}_\mu = \exp(\omega/2)l_\mu$. It is then easily seen that the stress-energy tensor in the reduced theory in eq.(19) has the same functional form of that in the original theory, with the “renormalised” energy density and charge

$$\rho \rightarrow \bar{\rho} = \exp(-2\omega)\rho \quad e^2 \rightarrow \bar{e}^2 = \exp(-2\omega)e^2. \quad (24)$$

The solutions of the new field equations (16) can therefore be classified by looking at the minima of the potential term (15). Even if in the original theory a cosmological term is not present, the solutions of the reduced theory are given by a Vaydia like spacetime in a de Sitter (dS) or Anti-de Sitter (AdS) background, depending on the value of the potential at minimum. In particular, one can use the Eddington-Finkelstein coordinates so that an explicit form of the metric element in the reduced theory reads

$$d\bar{s}^2 = dv(2dr - f dv) + r^2 d\Omega^2. \quad (25)$$

where

$$f = 1 - \frac{2m(v)}{r} + \frac{\bar{e}^2}{r^2} - \frac{\Lambda}{3}r^2 \quad (26)$$

is the so called mass function, $\Lambda = V(\omega_c)$ and ω_c is the location of any extrema of the potential. In a local chart adapted to the inner horizon the Cauchy horizon is located at $v = +\infty$, and $f = 0$. If we compare this metric with the solution we would have gotten in the Einstein gravity, we note that, according to eq.(24) the charge and the energy densities are screened or anti-screened depending on the sign of ω_c . It should be also noted that, while in standard gravity one has always AdS solution of the kind (25) in our case the above metric is a solution of the field equations only for special values of the cosmological constant.

The global spacetime structure of those solution is well known. An updated review can be found in the contribution by Chris Chambers in these proceedings, while recent results on topological (AdS) black holes can be found in the lecture of R. Mann [28]. See also there for the figures displaying the conformal structure of the two spacetimes.

Here we shall simply stress that the black hole spacetime in both dS and AdS solutions may have Cauchy horizons depending on the roots of the equation

$$f(r) = 0 \quad (27)$$

If the dS case is considered eq.(27) may have three positive roots and the spacetime has therefore a cosmological horizon besides an inner and an event horizon. Thanks to the work of Brady and Poisson [29] we know the Cauchy horizon is stable if the surface gravity of the cosmological horizon is greater than the surface gravity of the inner horizon

$$\kappa_{CS} > \kappa_{CH} \quad (28)$$

The surface gravity of the horizon in the original theory is obtained by a simple scale transformation (from the definition of surface gravity), and one can conclude that the stability criteria (28) holds in the original theory as well.

However, a black hole in dS background does not represent a vacuum solution since the field sets on a maximum of the potential. The stable vacuum solutions for the excitations of the ω field are instead given by black holes in AdS background and for those solutions the Cauchy horizon is a subtle question. The spatial infinity is now timelike, the radiative falloff at late times is not well understood [30]. For a power law decay of the kind

$$\delta(v) = 1/(\alpha v)^p \quad (29)$$

with $p \geq 3$ and α has to be introduced on dimensional grounds, the Cauchy horizon is unstable. In fact in general the energy density ρ_{obs} measured by an observer that crosses the Cauchy horizon would diverge as

$$\rho_{obs} \sim \dot{\delta} \exp(2\kappa_{CH}v) \quad (30)$$

thus signalling the instability of the Cauchy horizon. The CH in the original theory would also be unstable since the conformal transformation in eq. (11) is regular. However for an exponential decay rate of the kind

$$\delta \sim \exp(-2\alpha v) \quad (31)$$

one would not see any divergence for

$$\alpha > \kappa_{CH} \quad (32)$$

In a more realistic situation, one should use the eq. (29) to mimic the radiation falling into the event horizon. Outside the event horizon there is no reason to suppose that the background being AdS. Our effective “cosmological” constant is therefore dynamically generated during the gravitational collapse for the presence of the quantum fluctuations when the curvature has reached planckian level. The initial data coming from larger radii is still given by the classical evolution and therefore the Cauchy horizon would still be unstable even in the presence of a negative (or positive) Λ term in the metric.

III. R^2 , $R^3 + R^2$ GRAVITY

Let us analyse some cases in detail. If we consider R^2 gravity, the density Lagrangian of the gravitational field reads

$$l_g(R) = \epsilon_0 + \epsilon_1 R + \frac{1}{2}\epsilon_2 R^2. \quad (33)$$

then, eq. (12) can be inverted to yield $\psi = \ln(1/2 + \epsilon_2 \psi)$ provided $\psi > -1/2\epsilon_2$. In particular the potential term becomes

$$V(\omega) = \frac{1}{2\epsilon_2}(1 - \epsilon_1 e^{-\omega})^2 - e^{-2\omega}\epsilon_0 \quad (34)$$

We see that for a positive cosmological constant in the original theory the potential term is not bounded from below and there are no stable solutions of the quadratic gravity theory. In the case of a negative cosmological constant, the potential term has only one minimum for $\omega_c = -\ln 2$ and we find the BH solutions given by eq.(26). We note that the “renormalised” charge and energy density have now an increased strength of a factor 4 according to (24).

Another interesting case is that of R^3 gravity where we suppose that the original Lagrangian has the following functional form

$$l_g(R) = \epsilon_1 R + \frac{1}{2}\epsilon_2 R^2 + \frac{1}{3!}\epsilon_3 R^3 \quad (35)$$

in this case we can invert the relation (12) in order to obtain the two solutions

$$\psi_{\pm}(\omega) = \frac{1}{\epsilon_3} \left(-\epsilon_2 \pm \sqrt{\epsilon_2^2 + 2(e^\omega - \epsilon_1)\epsilon_3} \right) \quad (36)$$

in the two branches $\psi > -\epsilon_2/\epsilon_3$ and $\psi < -\epsilon_2/\epsilon_3$ and for the reality condition we also consider $\omega > \omega_* = \ln(1/2 - \epsilon_2^2/\epsilon_3)$. The potential term is therefore given by

$$V(\psi_{\pm}(\omega)) = \frac{\epsilon_2}{2}\psi_{\pm}^2(\omega)e^{-2\omega}\left(1 + \frac{2\epsilon_3}{3\epsilon_2}\psi(\omega)\right) \quad (37)$$

In the first case the potential $V(\psi_+(\omega))$ has one minimum for $\omega_c = -\ln 2$ with $V(\omega_c) = 0$ and a local maximum at another $\omega_c < 0$ with $V(\omega_c) > 0$. Therefore in the ψ_+ branch we can conclude that the solution of the field equations is given by a Vaidya like metric in a dS background. In the ψ_- branch we have only one local minimum that corresponds to a negative value of the potential, and therefore the metric is still a Vaidya like, but in AdS background.

Other interesting cases are given by Lagrangian like $l_g(R) = R + bR \ln(a + R)$, with a and b constants, which would arise from a one-loop contribution in an interacting stress-energy tensor. In any case one can use the above outlined procedure, and derive the explicit expression of the dS or AdS black hole solution.

The use of the conformal transformation is not the only method to obtain the solution of the original theory. There is also a direct procedure which is of course equivalent. Let us consider the field equations given in (8) and set

$$R_{\mu\nu} = C(R)(\rho l_{\mu}l_{\nu} + E_{\mu\nu}) + \frac{1}{4}Rg_{\mu\nu} \quad (38)$$

with constant Ricci scalar R . The following statement is true:

The solutions of the field equations in (8) with the stress energy-tensor in (20) and the Ricci tensor of the form (38) have

$$Rl'_g(R) - 2l_g(R) = 0 \quad (39)$$

$$l'_g(R)C(R) = 1. \quad (40)$$

The above conditions have to be considered as implicit relations that constrain the possible values of the constant R . They can be deduced by direct substitution of (38) in (8) and by projection along $l^{\mu}n^{\nu}$ and $n^{\mu}n^{\nu}$ respectively. One also notes that the first of the above relations in (38) is equivalent, aside from a constant factor, to the extrema condition for the potential, and the second relation can be read off as $C(R) = e^{-\omega}$ which is the first of the relations in (11). This shows the equivalence of the two approaches in obtaining solutions of the HD theory. However the conformal transformation method is simpler and elegant, while the direct method has the advantage that can also be used when $l''_g(R) = 0$. The set of consistency equations in (38) can also be deduced by considering the following stress energy tensor

$$T_{\mu\nu} = C(R)^{-1}\rho l_{\mu}l_{\nu} + E_{\mu\nu} \quad (41)$$

and with the Ricci tensor given by

$$R_{\mu\nu} = \rho l_{\mu}l_{\nu} + C(R)E_{\mu\nu} + \frac{1}{4}Rg_{\mu\nu}. \quad (42)$$

The above relations can be used to investigate the full global content of the theory. Once eq.(38) has been solved, one introduces locally the conformal frame where the new degrees of freedom is made explicit. Then one can address the question of local stability and mass of the field.

IV. CONCLUSIONS

As we have stressed in the introduction, the non-linear modifications of the standard Einstein theory are needed for taking into account of the quantum effects when the Weyl curvature has reached Planckian levels. We have seen that stable black hole solutions appears in these cases. What is the possible role of these new solutions in the standard mass inflation scenario? It is interesting to understand the structure of the CH singularity in a more consistent calculation where these new terms are dynamically generated during the collapse. We have argued that the role of those new terms would not change the (unstable) character of the Cauchy horizon. We therefore think that the mass inflation phenomenon also takes place in such a background. However since the appearance of an effective Λ term changes both the location of the horizon and the surface gravity of the inner horizon, a detailed calculation must be performed in order to address this issue.

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